Black And White Bin Packing Revisited

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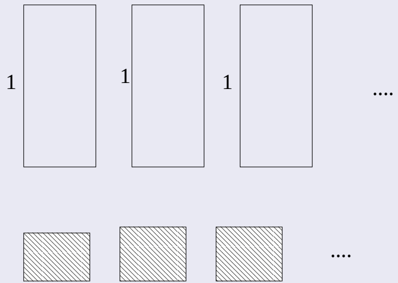
Outline

Bin Packing Problem

The input



Output: pack all the items into bins to minimize the number of bins used.

online Bin Packing Problem

Online vs Offline

- offline: before making decision, all info of n items are given.
- online: item is given one by one, and you cannot change previous decisions.

Evaluating Online Algorithms

Competitive Ratio

$$C_A^\infty = \lim_{n\to\infty} \sup_L \{A(L)/OPT(L) \mid OPT(L) = n\}.$$

Online bin packing

Previous results

- Lower bounds: 1.5 $\rightarrow$ 1.54017 $\rightarrow$ 1.54037, [2012TCS].
- Upper bounds: 2 $\rightarrow$ 1.7 $\rightarrow$ 1.69103 $\rightarrow$ 1.666$\rightarrow$ 1.58889 [J.ACM 2002].

A new model: online black white bin packing

properties

- size : (0, 1];
- colors: black, white;

Constraints

- Total size is at most 1;
- Two items with the same colors cannot be packed together;
- Input is online.

Target

- Min the total number of bins used.

Offline B-W bin packing

Input

- sizes: 0,0,0,0,.....
- Colors: bbbbbbbbbbb...., wwwwww.....

Two kinds of offline algorithms

- Full offline: one bin is enough.
- Restricted Offline: packing has to be according to L, in contrast to the online situation, the sizes and colors are known in advance. So, *n* bins are used.

Evaluating Online B-W Algorithms

Competitive Ratio

$$C_A^{\infty} = \lim_{n \to \infty} \sup_L \{A(L)/OPT(L) \mid OPT(L) = n\},$$

where OPT stands for the restricted offline optimal algo.

Online B-W bin packing

Previous results: two colors

- Lower bounds: 1.732 $\rightarrow$ 2 [2015].
- Upper bounds: 3 ($1 + \frac{d}{d-1}$ if the larger item is $\frac{1}{d}$)[2012].

Previous results: $C \geq 3$ colors

- Lower bounds: 2.5 [2014].
- Upper bounds: 4 (absolute), 3.5 (Asymptotic) [2014].
- lower bound = upper bound = 1.5 if all items have size zero [2014].

Online B-W bin packing

Previous results: two colors

- FF, WF are 3-compeitive [2014].
- Pseudo is also 3-compeitive[2015].

Previous Online Algorithms

Pseduo

- Stage 1: ignore sizes, or, view sizes to zero. According colors, pack items into stacks.
- Stage 2: in each stack, call NF to items into bins.

An example

Pseduo

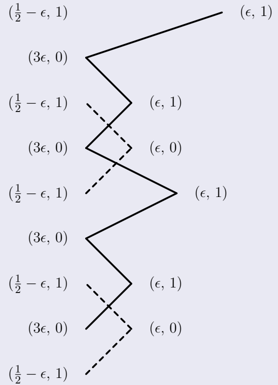


Figure: Worst Case of Pseudo Algorithm for items of sizes $(0, 1/2]$

Even all the items are at most 1/2, the best known algo has

An example

Pseduo

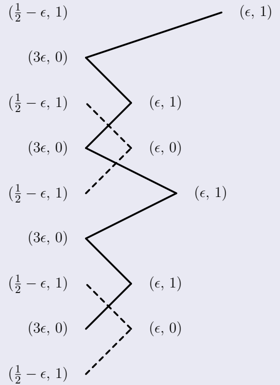


Figure: Worst Case of Pseudo Algorithm for items of sizes $(0, 1/2]$

Even all the items are at most 1/2, the best known algo has

Worst case of FF

competitve ratio at least 3

- first list L1 contains N white items with size $1 - 3\epsilon$ (as 0,0,...0, where 0 denotes white item),
- then L2 contains N pairs, where each pair have one black item with size 2ϵ (as 1,0,1,0,...1,0, where 1 denotes black item),
- next L3 contains N pairs of one black item followed one white item where both items have size ϵ (as 1,0,...,1,0),
- finally list L4 contains N black items of size ϵ (as 1,1,...,1).

Worst case of zero size, and $c \geq 3$

lower bound of optimal solution

Formally, let $s_{c,i} = 1$ if the i-th item from the input sequence has color c, and $s_{c,i} = -1$ otherwise. We define

$$LB_2 = \max_{c \in C} \max_{i,j} \sum_{k=i}^{j} s_{c,k}.$$

lower bound 1.5

Worst case of zero size, and $c \geq 3$

upper bound 1.5

Online Algorithm, and $c \geq 3$

upper bound

Motivation

Target

Target: to beat the upper bound 3.

Worst case of Pesudo

high vs flat

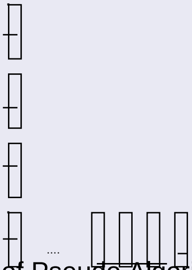


Figure: Worst Case of Pseudo Algorithm

high vs flat

- In the high stack, each bin has volume near 0.5
- In the flat stack, each bin has volume near 0.

Start points

Key ideas

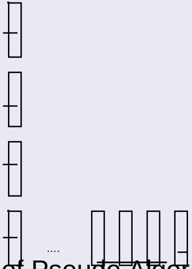


Figure: Worst Case of Pseudo Algorithm

Try to merge flat stacts into high stack. How?

Worst case 1

high stack first

Figure: Worst Case of Pseudo Algorithm

Worst case 2

Flat stacks first

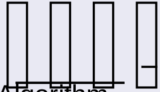


Figure: Worst Case of Pseudo Algorithm

High stack first

Couple pair

- Two bins
- Total size larger one
- Different top colors

Worst case 1

high stack first

Figure: Worst Case of Pseudo Algorithm

Flat stacks first

Couple pair

- Two bins
- Total size larger one
- Different top colors

Worst case 2

flat stack first

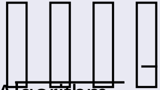


Figure: Worst Case of Pseudo Algorithm

Future work

- Improve upper bound 3 for black-white bin packing without any constraint.
- Improve upper bound 3.5 for colored bin packing.

Ends

Thanks.